

## **Title: Peak Performance**

### **Brief Overview:**

Students will construct trigonometric models of the three biorhythmic cycles. Students will predict when the three cycles and the combinations will simultaneously peak. Students will use their models to verify or refute these predictions.

### **NCTM Content Standards/National Science Education Standards:**

#### **National Council of Teachers of Mathematics Content Standards**

##### **Number and Operations**

- Students will understand numbers, ways of representing numbers, relationships among numbers, and number systems.

##### **Algebra**

- Students will understand patterns, relations, and functions.
- Students will use mathematical models to represent and understand quantitative relationships.
- Students will analyze change in various contexts.

##### **Data Analysis and Probability**

- Students will formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.
- Students will select and use appropriate statistical methods to analyze data.
- Students will understand and apply basic concepts of probability.

### **Process Standards:**

##### **Problem Solving**

Students will be able to build new mathematical knowledge through problem solving.

##### **Communication**

Students will be able to organize and consolidate their mathematical thinking through communication.

##### **Connection**

Students will be able to understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

##### **Representation**

Students will be able to use representation to model and interpret physical, social, and mathematical phenomena.

**National Science Education Standards:****Unifying Concepts and Processes**

- Students will demonstrate the relationship between logic, evidence, and current knowledge.

**Science as Inquiry**

- Students will use technology and mathematics to improve investigations and communications.

**Science and Technology**

- Students will communicate the problem, process, and solution.
- Students will present results to students, teachers, and others in a variety of ways, such as orally, in writing, models, and diagrams.

**Maryland High School Mathematics Core Learning Units****Functions and Algebra**

- 1.1.3 The student will apply addition, subtraction, multiplication, and or division of algebraic expressions to mathematical and real-world problems.
- 1.1.4 The student will describe the graph of a non-linear function.
- 1.2.5 The student will apply formulas to solve real-world problems.

**Grade/Level:**

Lessons 1, 2, and 3: Grades 9-12; Algebra II/Trigonometry, Pre-Calculus

**Duration/Length:**

Two to three class periods, each approximately 50 minutes in length.

**Student Outcomes:**

Students will:

- Be able to construct and make inferences from sine graphs.
- Be able to create a mathematical model of a periodic function.
- Be able to use the TI-83 calculator to analyze data from real world models.
- Be able to algebraically transform functions (phase shifts).

**Materials and Resources:**

- TI-83 Calculator
- Worksheets
- Assessment Sheets
- Teacher Fact Sheet

## **Biorhythm Facts**

The theory of biorhythms, which has been around since the end of the nineteenth century, claims that our lives are affected by rhythmic cycles. Supposedly, the events in all people's lives are dictated by three principal biorhythmic cycles: the physical cycle (23 days), the emotional cycle (28 days), and the intellectual cycle (33 days). Although the theory was especially popular in the 1970s, the cycles of biorhythm theory have never been supported by a credible scientific study.

A person's location in his or her biorhythmic cycles is purported to predict what kind of day one is likely to have. Generally speaking, the more positive a cycle is on any given day, the better one is able to interact in that arena. For example, a high intellectual level means that a person would be more adept at problem solving, test taking, or other intellectual pursuits.

At the moment of birth, all three of a person's biorhythmic cycles are set to zero. Knowing your birthday and the number of days you have lived enables you to determine where you are in each cycle on any given day. A cycle is said to be in a positive phase when its value is greater than zero, and in a negative phase when its value is less than zero. For our purposes, it is convenient to assume that the maximum value achieved in any cycle is +1, and the negative value achieved in any cycle is -1, although this designation is arbitrary. The cycles repeat until you die. Should you live to be 58 years and 66 days old, all cycles will return to zero simultaneously – the so-called moment of “rebirth”.

Anecdotal evidence has been given to support biorhythms, but of course it would be just as easy to find other evidence which would refute the theory. For example, defenders of the theory point out that Mark Spitz (born 2/10/1950) was in a high physical and emotional phase when he won seven gold medals in the 1972 Olympics. On the other hand, Reggie Jackson had the greatest day of his career on October 18, 1977, when he hit three consecutive home runs on three consecutive pitches off three different pitchers to help the New York Yankees win the game and the World Series against the Los Angeles Dodgers. All three of Jackson's biorhythmic cycles were on the low end of their negative phases on that day.

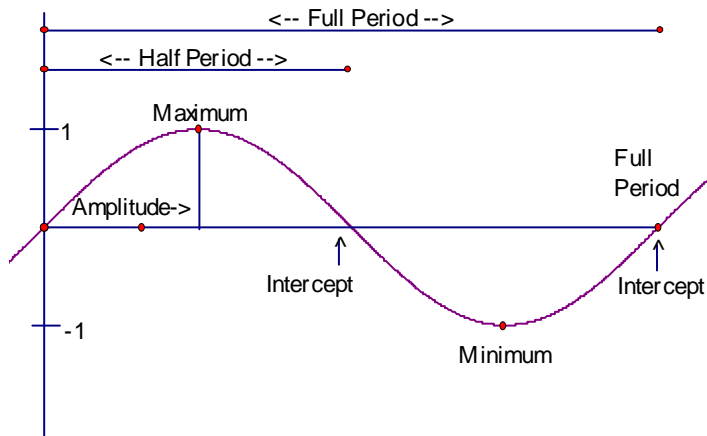
Source: Skeptic's Dictionary at [www.skepdic.com](http://www.skepdic.com). (June 30, 2003)

## Development/Procedures:

### Lesson 1

#### Preassessment :

$$f(x) = \sin(x)$$



- 1) What is the value of the amplitude? \_\_\_\_\_
- 2) What is the maximum value of the function? \_\_\_\_\_
- 3) The above graph of  $\sin(x)$  represents one or two periods? \_\_\_\_\_

For each set of values  $x, y, z$ , for which  $z = x/y$ , find the unknown.

- 4)  $x = 2\pi$ ,  $y = 4$ ,  $z = ?$
- 5)  $x = 3\pi$ ,  $y = 60$ ,  $z = ?$
- 6)  $x = 4\pi$ ,  $y = 30$ ,  $z = ?$
- 7)  $x = ?$ ,  $y = 4$ ,  $z = 3\pi/4$ ?
- 8)  $x = 2\pi$ ,  $y = 3/2$ ,  $z = ?$
- 9)  $x = 3\pi$ ,  $y = 3/5$ ,  $z = ?$
- 10) What is the distinction between a cycle and a period of a trigonometric function?  
\_\_\_\_\_  
\_\_\_\_\_

### Launch

- From a scale of -1 to 1; the students should draw a graph of their physical, emotional, and intellectual status for the past month. (Teacher Note: this could be done prior to the start of the lesson)
- The teacher will review and discuss the pre-assessment.

### Teacher Facilitation

(Teacher Note) Biorhythm Fact Sheet

- The teacher will discuss the biorhythmic cycles: physical (23), emotional (28), intellectual (33)
- The students will first calculate their specific age by days (Student Reminder: include leap years)

### Student Application

Let  $p(x) = 1 \sin(bx)$  (Physical)  
 $e(x) = 1 \sin(bx)$  (Emotional)  
 $i(x) = 1 \sin(bx)$  (Intellectual)

Period =  $2\pi/b$ , where  $b$  is the period of the biorhythmic cycle, graph the following:

$p(x) = \sin(2\pi x/23)$   
 $e(x) = \sin(2\pi x/28)$   
 $i(x) = \sin(2\pi x/33)$

(Teacher Note)

Settings:

Mode: Radian

Window:

Xmin = 15 days less than actual "days old"

Xmax = 15 days more than actual "days old"

Xscl = 1

Ymin = -1

Ymax = 1

Yscl = 1

Xres = 1

### Embedded Assessment

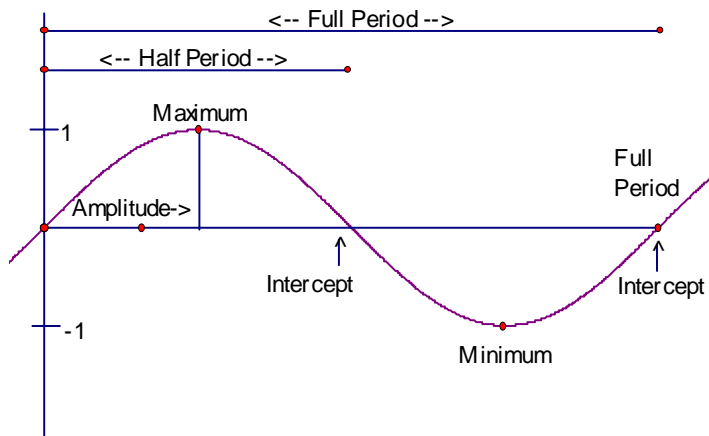
Students will exchange actual birth dates and calculate the number of days old of a partner. Students will compute one of the biorhythms and illustrate the graph. A comparison of the graph will be matched with each partner. (Both partners should graph the same biorhythm).

### Reteaching/Extension

- The teacher should create small groups for lesson review.
- The teacher should review any needed material/information.
- The students will write and explain the following:  
“What do you think would happen to the sine graph of the combination of two biorhythms?”

## Lesson 1 (Pre-Assessment Answer Key)

$$y = \sin(x)$$



- 1) What is the value of the amplitude? **1**
- 2) What is the maximum value of the function? **1**
- 3) The above graph of  $\sin(x)$  represents one or two periods? **1**

For each set of values  $x, y, z$ , for which  $z = x/y$ , find the unknown.

4)  $x = 2\pi$ ,  $y = 4$ ,  $z = ?$       **$\pi/2$**

5)  $x = 3\pi$ ,  $y = 60$ ,  $z = ?$       **$\pi/20$**

6)  $x = 4\pi$ ,  $y = 30$ ,  $z = ?$       **$2\pi/15$**

7)  $x = ?$ ,  $y = 4$ ,  $z = 3\pi/4$ ?      **$3\pi$**

8)  $x = 2\pi$ ,  $y = 3/2$ ,  $z = ?$       **$4\pi/3$**

9)  $x = 3\pi$ ,  $y = 3/5$ ,  $z = ?$       **$5\pi$**

- 10) What is the distinction between a cycle and a period of a trigonometric function?

**Each function is periodic; the graph has a pattern that repeats infinitely. The shortest repeating portion is a cycle. The horizontal length of each cycle is the period.**

## **Development/Procedures:**

Lesson 2

### Preassessment

Student Activity Sheet – Question #1

### Launch

Determine your peak performance dates for this school year and for future events.

### Teacher Facilitation

Refer to Teacher Worksheet

### Student Application

Lesson 2 Activity Sheet

### Embedded Assessment

Built into student worksheet

### Reteaching/Extension

- Teacher should create small groups for lesson review
- The teacher should review any needed material/information
- The students should reflect upon the following:  
“How would you go about finding a peak performance date for a group of people?” (The group could be a starting 5 on a basketball team).



## Peak Performance – Lesson 2

### Student Worksheet

Name: \_\_\_\_\_

1. Graph the following biorhythm cycles on your graphing calculator. Be sure to show the cycles pertaining to your first 100 days since birth. Check with your neighbor to ensure that you have the correct cycles. Draw and label a sketch of your physical cycle below.

Y1 – The graph of your 23 day physical cycle

Y2 – The graph of your 28 day emotional cycle

Y3 – The graph of your 33 day intellectual cycle

2. a) Determine how many days old you were on the first day of school this year. Remember to take into account extra days during a leap year. Show work below. A written explanation may be helpful.

Date of first day of school \_\_\_\_\_ Age in days \_\_\_\_\_

2. b) Determine how many days old you are today. Use the table on your graphing calculator to determine the value of your three biorhythmic cycles today.

Date today \_\_\_\_\_ Age in days \_\_\_\_\_

Value of physical cycle \_\_\_\_\_ emotional cycle \_\_\_\_\_ intellectual cycle \_\_\_\_\_

3. Use your calculator to find the dates of your peak performance in each biorhythm cycle during this school year. List these dates below.

Physical \_\_\_\_\_

Emotional \_\_\_\_\_

Intellectual \_\_\_\_\_

4. Your passion cycle is a combination of your physical cycle joined with your emotional cycle. Knowing this, make a prediction date of when your passion cycle will peak during this school year. Explain your reasoning.

Passion Peak \_\_\_\_\_

5. What do you think the amplitude of your passion cycle will be? Explain your reasoning.

Amplitude \_\_\_\_\_

6. Would you expect the passion cycle to be periodic? Explain.

7. If you answered “yes” in Number 6, what would you expect the period to be? Explain your reasoning.

8. Do you believe the graph of the passion cycle will be sinusoidal like the graphs of the physical cycle and emotional cycle? Explain your reasoning

9. Draw the graph of what you believe the passion cycle might look like. Explain your reasoning for choosing this depiction for as your graph.

10. Using your knowledge of addition of functions, what algebraic model might you use to describe the passion cycle?

11. Using the model developed in question #10, graph the first 100 days of your passion cycle on your calculator. Draw and label a sketch of this graph below.

12. Are you surprised by the results of the graph? Explain why or why not and include similarities and differences between this graph and your guess from question #9.

13. Use your passion model and your calculator, to find on what day of this school year you will produce your “peak performance” in passion. Explain how you determined the peak performance.

14. Your wisdom cycle is a combination of your emotional and intellectual cycles and your mastery cycle is a combination of your intellectual cycle and physical cycle. Write the equation for each of these Biorhythm cycles below.

Wisdom cycle \_\_\_\_\_

Mastery cycle \_\_\_\_\_

15. Use your wisdom and mastery models and your calculator, to find on what day of this school year you will produce your “peak performance” in each of these areas.

Peak performance wisdom \_\_\_\_\_

Peak performance mastery \_\_\_\_\_

16. Your biorhythmic cycle consists of your physical, emotional and intellectual cycles. Write a model below that makes one biorhythmic model that encompasses all 3 cycles.

17. What is the amplitude of this model? Explain how you determined the amplitude.

18. Find the period of the model and explain how you found it. If there is no period, explain why there is not one.

19. List 5 important events that you believe will happen in your future. Use your calculator and your knowledge of biorhythms to determine future dates for these events that coincide with your “peak performance”.

## Peak Performance – Lesson 2

### Teacher Worksheet

Name: \_\_\_\_\_

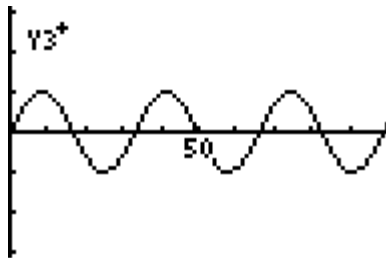
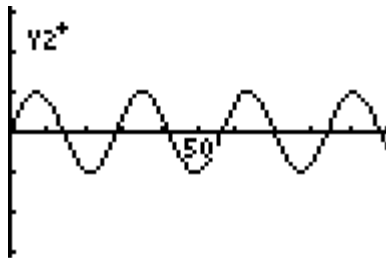
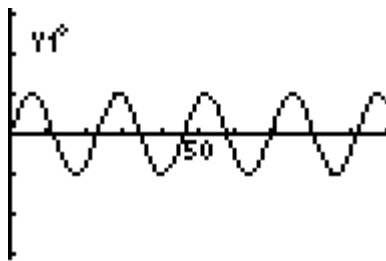
1. Graph the following biorhythm cycles on your graphing calculator. Be sure to show the cycles pertaining to your first 100 days since birth. Check with your neighbor to ensure that you have the correct cycles. Draw and label a sketch of your physical cycle below.

Y1 – The graph of your 23 day physical cycle

Y2 – The graph of your 28 day emotional cycle

Y3 – The graph of your 33 day intellectual cycle

Circulate around the room and check student graphs with those below.



2. a) Determine how many days old you were on the first day of school this year. Remember to take into account extra days during a leap year. Show work below. A written explanation may be helpful.

**Date of first day of school : August 27      Age in days: 5905 days**

**These answers based on a person who is 16 on August 27<sup>th</sup> and the current date is October 27<sup>th</sup> (61 days later)**

2. b) Determine how many days old you are today. Use the table on your graphing calculator to determine the value of your three biorhythmic cycles today.

**Date today: October 27**

**Age in days: 5966 days**

**Value of physical cycle: 0.631      emotional cycle: 0.434      intellectual cycle: -0.972**

3. Use your calculator to find the dates of your peak performance in each biorhythm cycle during this school year. List these dates below. Be sure to have a range that corresponds to your number of days since birth for the start of school and for the end of school.

**Answers based on b-day used in #2**

**Physical: about 11-12 peak days - including day 5985 or January 15th**

**Emotional: about 9-10 peak days - including day 5971 or January 1st**

**Intelligence: about 8 peak days - including day 5982 or January 12th**

4. Your passion cycle is a combination of your physical cycle joined with your emotional cycle. Knowing this, make a prediction date of when your passion cycle will peak during this school year. Explain your reasoning.

Passion Peak **Answers will vary**

**You may want to discuss Student responses.**

5. What do you think the amplitude of your passion cycle will be? Explain your reasoning.

Amplitude: **Two. This is the sum of the amplitudes of the two individual graphs.**

**You may want to discuss Student responses.**

6. Would you expect the passion cycle to be periodic? Explain.

**Answers may vary. Yes, because the 2 graphs comprising this cycle are periodic.**

**You may want to discuss Student responses.**

7. If you answered “yes” in Number 6, what would you expect the period to be? Explain your reasoning.

**I would expect the period to be 644 days because that is the LCM of the two cycle lengths.**

**You may want to discuss Student responses.**

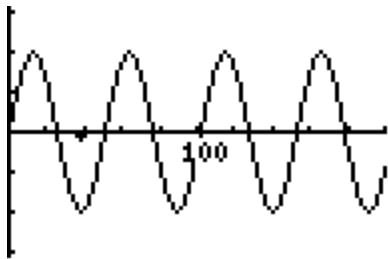
8. Do you believe the graph of the passion cycle will be sinusoidal like the graphs of the physical cycle and emotional cycle? Explain your reasoning

**Answers may vary. Yes, because the 2 graphs comprising this cycle are sinusoidal.**

**You may want to discuss Student responses.**

9. Draw the graph of what you believe the passion cycle might look like. Explain your reasoning for choosing this depiction for as your graph.

**Answers may vary. The graph below is a prediction of the first 200 days of the passion cycle. The amplitude is twice that of a single biorhythm cycle since you are combining 2 cycles. The period is now 51 since you are combining the 23 day period of the physical cycle and the 28 day cycle of the emotional cycle.**





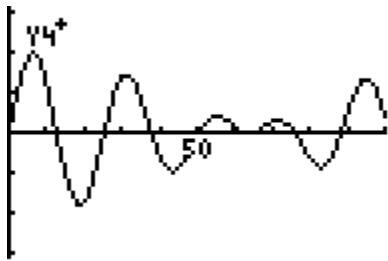
**You may want to discuss Student responses.**

10. Using your knowledge of addition of functions, what algebraic model might you use to describe the passion cycle?

**You may want to discuss Student responses.**

$$y = \sin\left(\frac{2\pi}{23}\right)x + \sin\left(\frac{2\pi}{28}\right)x \text{ is the actual answer}$$

11. Using the model developed in question #10, graph the first 100 days of your passion cycle on your calculator. Draw and label a sketch of this graph below.



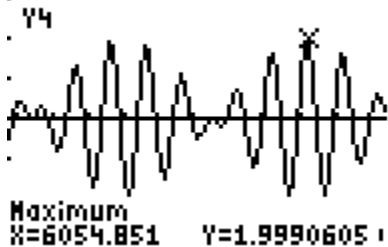
**Circulate and check students work. They should check their results with a neighbor.**

12. Are you surprised by the results of the graph? Explain why or why not and include similarities and differences between this graph and your guess from question #9.

**Answers may vary. Yes, I am surprised. I expected the graph to be a perfect sine curve. Both graphs have peaks and valleys and are smooth curves. The curve in number 10 is not predictable like the one in number 9.**

**You may want to discuss Student responses.**

13. Use your passion model and your calculator, to find on what day of this school year you will produce your “peak performance” in passion. Explain how you determined the peak performance



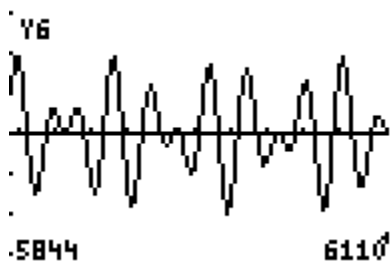
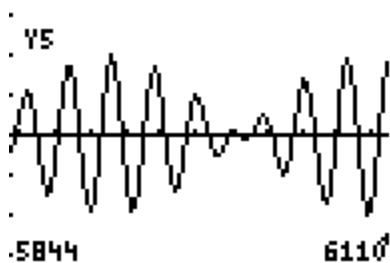
**March 25<sup>th</sup> according to example #2 data. I found the maximum point by using this feature on my calculator.**

14. Your wisdom cycle is a combination of your emotional and intellectual cycles and your mastery cycle is a combination of your intellectual cycle and physical cycle. Write the equation for each of these Biorhythm cycles below.

Wisdom cycle \_\_\_\_  $y = \sin\left(\frac{2\pi}{28}\right)x + \sin\left(\frac{2\pi}{33}\right)x$

Mastery cycle \_\_\_\_  $y = \sin\left(\frac{2\pi}{23}\right)x + \sin\left(\frac{2\pi}{33}\right)x$

15. Use your wisdom (Y5) and mastery (Y6) models and your calculator, to find on what day of this school year you will produce your “peak performance” in each of these areas. Show a sketch of each graph below. Please label appropriately.

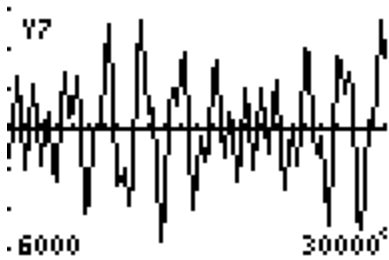


**Date of Peak performance wisdom: Max of 1.99 at 3<sup>rd</sup> peak from the left. The date will be November 6<sup>th</sup> of this school year.**

**Date of Peak performance mastery: Max of 1.92 at 2<sup>nd</sup> peak from the right. The date will be April 18<sup>th</sup> of this school year.**

16. Your biorhythmic cycle consists of your physical, emotional and intellectual cycles. Write a model below that makes one biorhythmic model that encompasses all 3 cycles. Graph the model below the equation.

$$y = \sin\left(\frac{2\pi}{23}\right)x + \sin\left(\frac{2\pi}{28}\right)x + \sin\left(\frac{2\pi}{33}\right)x$$



**This graph covers a range of 6,000 days to 30,000 days or 16.5 to 82 years of age.**

17. What is the amplitude of this model and explain how you determined the amplitude?

**The amplitude is 3. I found it by adding the amplitudes of the 3 original biorhythmic equations.**

18. Find the period of the model and explain how you found it. If there is no period, explain why there is not one.

**The period will be 21252 days (the LCM of the 3 periods) or about 58.18 years.**

19. List 3 important events that you believe will happen in your future. Use your calculator and your knowledge of biorhythms to determine future dates for these events that coincide with your “peak performance”. Explain your process for finding your answer. No two events should be on the same day.

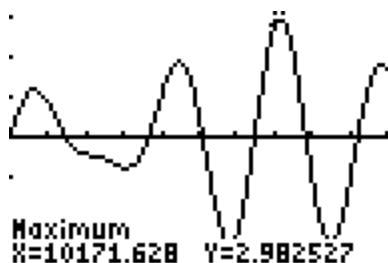
**Possible important events**

- 1. Marriage**
- 2. Job interview**
- 3. Major League Baseball tryout**
- 4. Run a marathon**
- 5. Test day for Law School, Medical School, and Graduate School etc.**

**Example: I want to get married between age 26 and 28. (9,496 to 10,227 days)**



**Focus on the spike on the right at 10,172 and adjust the window to 10,100-10,200.**



**The max of 2.98 occurs on day 10171. July 2nd after the person’s 27th birthday would be a peak point and a good day on which to get married.**

### Lesson 3: Group Biorhythms

#### Preassessment

In order to complete this lesson successfully, students should be familiar with how to transform the equations of functions so as to move their graphs to either the left or to the right.

That is, if the equation of the graph of  $y=f(x)$  is moved  $h$  spaces to the right, then its equation is given by  $y=f(x-h)$ . Similarly, if the equation of the graph of  $y=f(x)$  is moved  $h$  spaces to the left, then its equation is given by  $y=f(x+h)$ .

Preassessment questions:

- 1) The graph of  $y = \sin x$  is moved 3 units to the right. Find its new equation.  
(answer:  $y = \sin (x-3)$  )
- 2) The graph of  $y = \cos 2x$  is moved 5 units to the left. Find its new equation.  
(answer:  $y = \cos 2(x+5)$  )

#### Launch

Pose the following question to the students:

Suppose that the next test will not be an individual test, as most tests are, but instead is a group test. In this case, a group containing, say, three members, would want to take the test on a day when the “group” intellectual cycle is at a maximum. How would we go about determining an equation which models a *group’s* intellectual biorhythm?

#### Teacher Facilitation

Recall that to determine the value of one’s biorhythmic cycles for a given day, one needs to know which day of your life that day is. One does this by determining the number of days which have passed since your birthday.

Suppose we wanted to determine an equation which models a group’s intellectual biorhythmic cycle. We could create this new function by simply adding up the equations for each member of the group. We could then graph this new function for any period of time in order to determine the day at which it would peak.

However, there is one problem. When attempting to calculate an equation which models a “group” algorithm, one must consider that not everyone has the same birthday! In other words, on any given day, each group member will be on a different day of his or her life. We need to shift each individual’s equation so that there is a common starting day. *Then* we can add the equations together to determine a group algorithm.

So, what we need to do is to rewrite each individual’s equation so that all equations start from a common day. Any day will do, but a convenient common day is the birthday of the oldest person of the group. So what each group will need to do is to determine which group member is the oldest, and then determine how many days younger than the oldest member each other member is. Then, we can move the graph which models each group member’s intellectual cycle so that its cycle begins the appropriate number of days after the oldest member’s birthday (i.e., the graphs of the younger group members will be moved to the right on the graph).

For example, consider a group that has three members. Suppose that the oldest group member (call him Member A) was born on October 1, 1987. Suppose that another group member (call her Member B) was born on November 1, 1987. She would be 31 days younger than Member A. Then suppose that another group member (call her Member C) was born on December 1, 1987. She would be 61 days younger than Member A. The equations which model each member’s intellectual biorhythmic cycle are as follows:

$$\text{Member A: } i_a(x) = \sin\left(\frac{2\pi}{33}x\right)$$

$$\text{Member B: } i_b(x) = \sin\left(\frac{2\pi}{33}(x-31)\right)$$

$$\text{Member C: } i_c(x) = \sin\left(\frac{2\pi}{33}(x-61)\right)$$

Now, in order to create an equation which models each group’s intellectual cycle, one simply needs to add the equations for each group member:

$$i_g(x) = i_a(x) + i_b(x) + i_c(x).$$

In all of the equations,  $x$  represents the number of days since the birthday of the oldest group member (Member A).

### Student Application

Divide the students into groups. Have the students determine the oldest member of each group. Determine how many days younger than the oldest member each other member of the group is. Use this information to find one equation for each group member which models his or her intellectual biorhythmic cycle. Be sure to use a common day (the oldest group member's birthday) when determining these equations. Finally, add each member's equations together to obtain an equation which models the group's intellectual biorhythmic cycle.

These steps are summarized in the Lesson 3 worksheet which may be distributed to the students. Of course answers will vary depending on the birthdates of the members in each group, but sample answers are also provided.

### Embedded Assessment

Check on the progress of the groups as they progress through the Lesson 3 worksheet.

### Reteaching/Extension

One can repeat the procedures above for either the emotional or physical biorhythmic cycles. The procedure is the same as described above, except that the periods would be 28 days and 23 days for the emotional and physical cycles respectively.

## Peak Performance – Lesson 3: Group Intellectual Biorhythms

### Student Worksheet

Name: \_\_\_\_\_

1. Give an equation which models the 33 day intellectual biorhythmic cycle for  $x$  days after birth.

Equation:  $y =$  \_\_\_\_\_

2. Determine the age in days of each group member.

Member A is the oldest, Member B is the next oldest, and so on.

Then determine how many days younger than Member A each other group member is.

	Age in days	Days younger than Member A
Member A:	_____	_____
Member B:	_____	_____
Member C:	_____	_____
Member D:	_____	_____

3. Your answer to problem 1 models a person's intellectual biorhythmic cycle based on the days since his or her birthday. However, chances are that the members of your group will have different birthdays.

Thus, if we want to create an equation which models a group biorhythm, we will have to shift all equations to a common starting day. Give the equations for each group member, by shifting the graph given by Problem 1 by the number of days given in Problem 2.

Member A:  $y =$  \_\_\_\_\_

Member B:  $y =$  \_\_\_\_\_

Member C:  $y =$  \_\_\_\_\_

Member D:  $y =$  \_\_\_\_\_

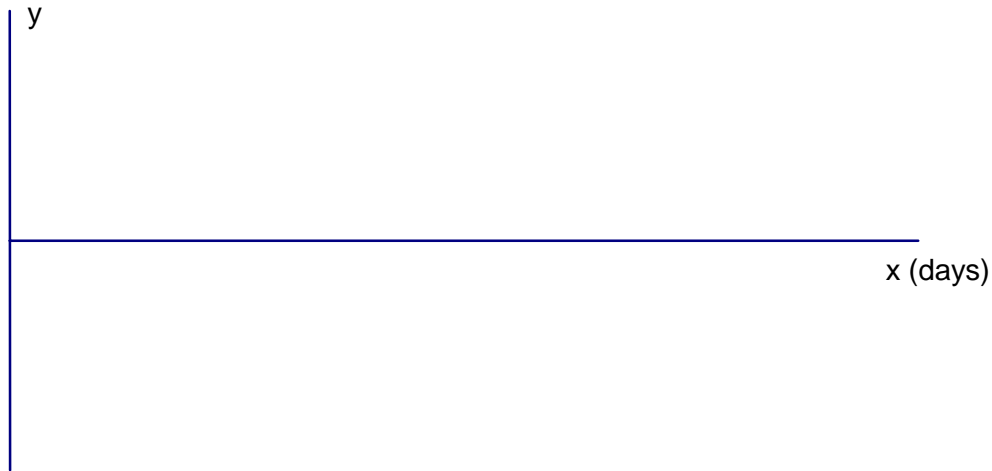


4. Find the number of days which have passed since Member A's birthday.

Number of Days: \_\_\_\_\_

5. Use your answer to Problem 4 to determine the correct values for your calculator's window so that your calculator displays the intellectual cycles for each group member for the 30 day period surrounding today's date (i.e., from fifteen days before today until fifteen days after today).

Graph Member A's cycle in Y1, Member B's cycle in Y2, and so on. Sketch these graphs below. Sketch all curves on the same set of axes, and label each curve with the appropriate person's name:



Give the date on which each member's intellectual biorhythmic cycle peaks during this 30 day period.

Member A: \_\_\_\_\_

Member B: \_\_\_\_\_

Member C: \_\_\_\_\_

Member D: \_\_\_\_\_

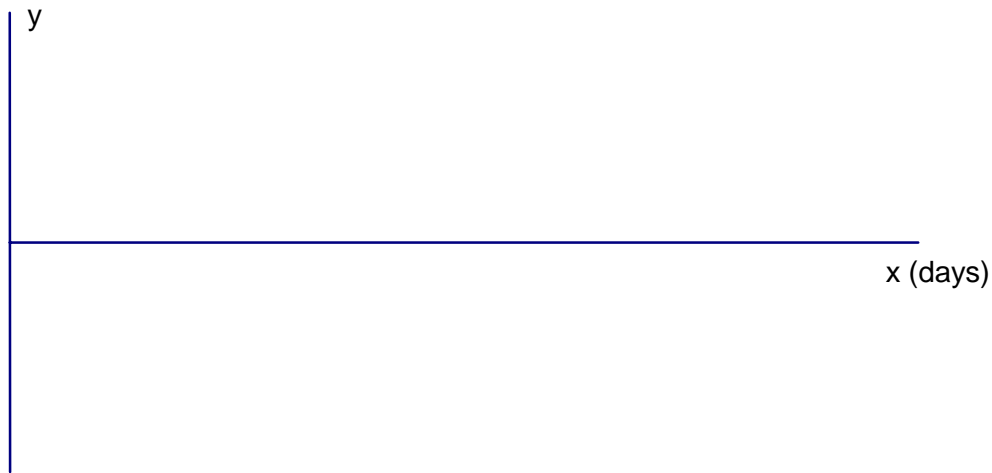
6. Now graph the group's intellectual biorhythmic cycle for the 30 day period surrounding today's date. Do this by adding together the equations for all group members.

For example, suppose that you have three group members. Member A's equation is in Y1, Member B's in Y2, and Member C's is in Y3. Then, in Y4, your equation should be  $Y1 + Y2 + Y3$ . The functions Y1, Y2, and Y3 can be found in the VARS menu on the TI-83.

Give an equation which represents the value of the group's intellectual biorhythm:

$y =$  \_\_\_\_\_

Sketch the graph below.



7. In the thirty day period surrounding today, on which day does your group's intellectual biorhythmic cycle reach its peak? How did you determine this?

8. Does the group's peak day occur on the same day as any individual's peak day? Does this surprise you? Explain why or why not?

### Peak Performance – Lesson 3: Group Intellectual Biorhythms

#### Student Worksheet

Name: KEY (answers may vary)

1. Give an equation which models the 33 day intellectual biorhythmic cycle for  $x$  days after birth.

Equation:  $y = \sin\left(\frac{2\pi}{33}x\right)$

2. Determine the age in days of each group member. Member A is the oldest, Member B is the next oldest, and so on. Then determine how many days younger than Member A each other group member is.

	Age in days	Days younger than Member A
Member A:	<u>5840</u>	<u>0</u>
Member B:	<u>5809</u>	<u>31</u>
Member C:	<u>5779</u>	<u>61</u>

3. Your answer to problem 1 models a person's intellectual biorhythmic cycle based on the days since his or her birthday. However, chances are that the members of your group will have different birthdays.

Thus, if we want to create an equation which models a group biorhythm, we will have to shift all equations to a common starting day. Give the equations for each group member, by shifting the graph given by Problem 1 by the number of days given in Problem 2.

Member A:  $y = \sin\left(\frac{2\pi}{33}x\right)$

Member B:  $y = \sin\left(\frac{2\pi}{33}(x - 31)\right)$

Member C:  $y = \sin\left(\frac{2\pi}{33}(x - 61)\right)$

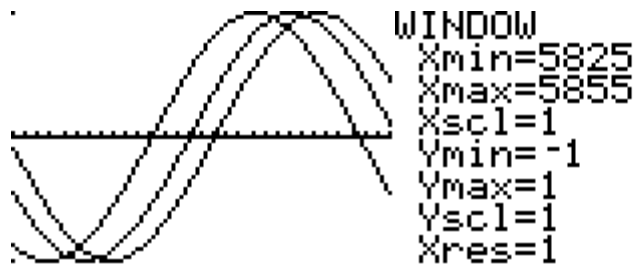
4. Find the number of days which have passed since Member A's birthday.

Number of Days: 5840

5. Use your answer to Problem 4 to determine the correct values for your calculator's window so that your calculator displays the intellectual cycles for each group member for the 30 day period surrounding today's date (i.e., from fifteen days before today until fifteen days after today).

Graph Member A's cycle in Y1, Member B's cycle in Y2, and so on. Sketch these graphs below. Sketch all curves on the same set of axes, and label each curve with the appropriate person's name:

*For these graphs, we have assumed today's date to be day 5840 of Member A's life.*



Give the date on which each member's intellectual biorhythmic cycle peaks during this 30 day period.

Member A: Day 5849 (9 days after today's date)

Member B: Day 5847 (7 days after today's date)

Member C: Day 5844 (4 days after today's date)

6. Now graph the group's intellectual biorhythmic cycle for the 30 day period surrounding today's date. Do this by adding together the equations for all group members.

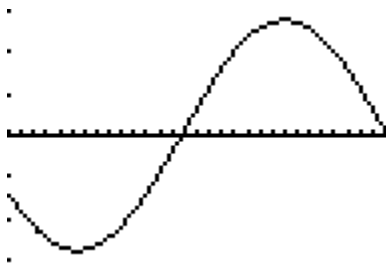
For example, suppose that you have three group members. Member A's equation is in Y1, Member B's in Y2, and Member C's in Y3. Then, in Y4, your equation should be  $Y1 + Y2 + Y3$ . The functions Y1, Y2, and Y3 can be found in the VARS menu on the TI-83.

Give an equation which represents the value of the group's intellectual biorhythm:

$$y = \sin\left(\frac{2\pi}{33}x\right) + \sin\left(\frac{2\pi}{33}(x-31)\right) + \sin\left(\frac{2\pi}{33}(x-61)\right)$$


---

Sketch the graph below.



```

WINDOW
Xmin=5825
Xmax=5855
Xscl=1
Ymin=-3
Ymax=3
Yscl=1
Xres=1

```

```

Plot1 Plot2 Plot3
Y1=sin((2π/33)X
)
Y2=sin((2π/33)(
X-31))
Y3=sin((2π/33)(
X-61))
Y4=Y1+Y2+Y3

```

7. In the thirty day period surrounding today, on which day does your group's intellectual biorhythmic cycle reach its peak? How did you determine this?

**Day 5847 (7 days after today's date)**

**Note that the window has been expanded to accommodate the increased amplitude of the graph.**

8. Does the group's peak day occur on the same day as any individual's peak day? Does this surprise you? Explain why or why not?

**The group peak occurs on the same day as Member B's peak day. This date makes sense in this case because it is three days after Member C's peak day but two days before Member A's peak day.**

### Summative Assessment:

1) New biorhythmic cycles have been added in recent years. There is a 38-day intuitional cycle, a 43-day aesthetic cycle, and a 53-day spiritual cycle. Write an equation which would model the value of your spiritual cycle on the  $x$ th day of your life. Give its amplitude and period.

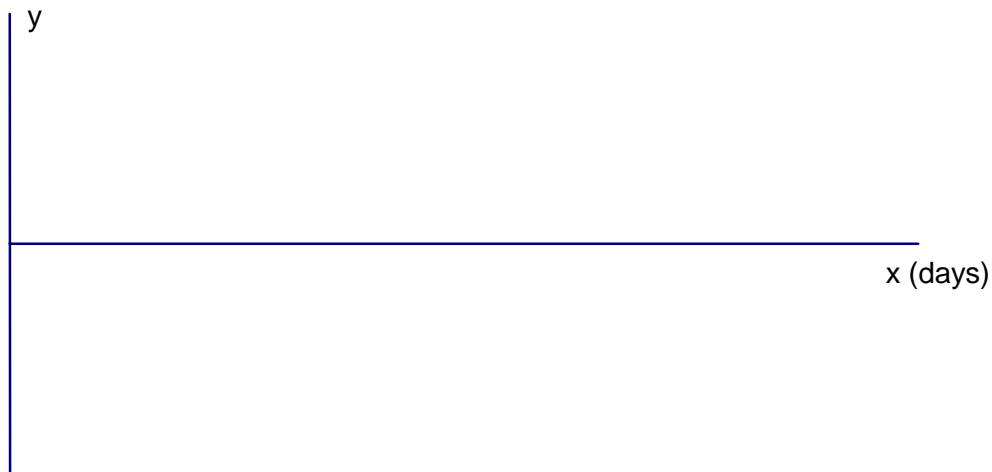
Equation: \_\_\_\_\_

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

2) Let's say that you are curious to see the value of your spiritual cycle on your 21<sup>st</sup> birthday. You determine that you will be 7670 days old on this day. Sketch a graph of your spiritual cycle for a period of 30 days surrounding that day (for 15 days before to 15 days after).

a) Sketch your graph on the axes below.

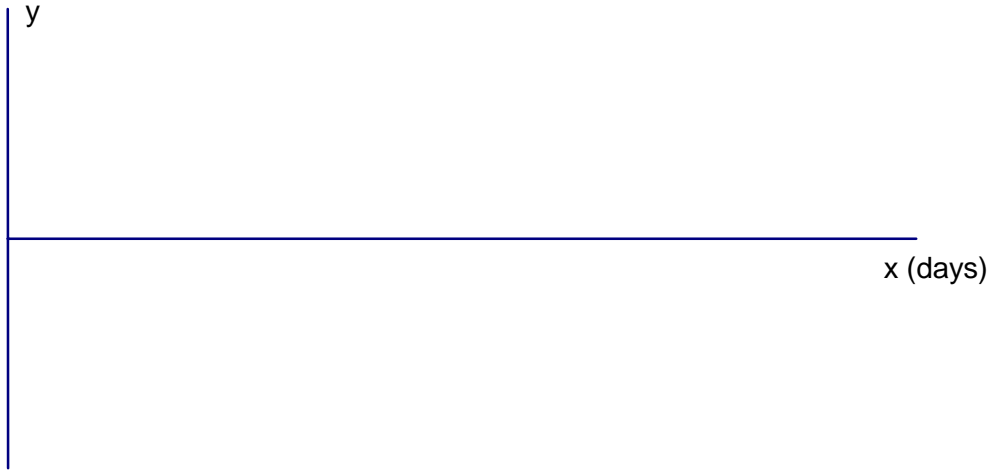


b) How many days before or after your 21<sup>st</sup> birthday does the spiritual cycle reach its minimum value?

3) Now, suppose that you wish to find the value of the sum of your spiritual cycle and your aesthetic cycle for the  $x$ th day of your life. Give an equation which models this sum.

Equation: \_\_\_\_\_

4) Now suppose that you would like to calculate the value of the sum of your spiritual cycle and your aesthetic cycle in the 30-day period surrounding your 21<sup>st</sup> birthday. Sketch your graph on the axes below.



5) Suppose that you have a friend who was born 90 days after you were. Write an equation which describes the value of his spiritual biorhythmic cycle for the  $x$ th day of *your* life.

Equation: \_\_\_\_\_

6) Now suppose that you would like to know the value of the sum of your spiritual cycle and your friend's spiritual cycle on the  $x$ th day of *your* life. Write an equation which models this sum.

Equation: \_\_\_\_\_

7) Use your calculator to graph your equation from Problem 6 for a period of 30 days surrounding your 21<sup>st</sup> birthday. How many days before or after your 21<sup>st</sup> birthday does this equation reach its minimum value?

### Summative Assessment: Answer Key

1) New biorhythmic cycles have been added in recent years. There is a 38-day intuitional cycle, a 43-day aesthetic cycle, and a 53-day spiritual cycle. Write an equation which would model the value of your spiritual cycle on the  $x$ th day of your life. Give its amplitude and period.

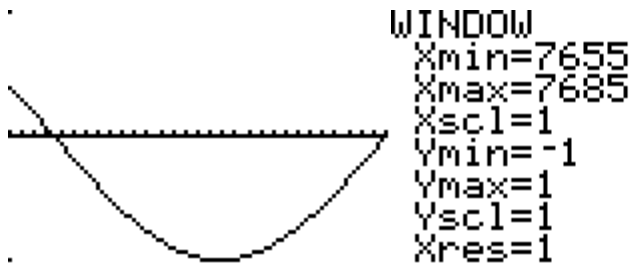
Equation:  $y = \sin\left(\frac{2\pi}{53}x\right)$

Amplitude: 1

Period: 53 days

2) Let's say that you are curious to see the value of your spiritual cycle on your 21<sup>st</sup> birthday. You determine that you will be 7670 days old on this day. Sketch a graph of your spiritual cycle for a period of 30 days surrounding that day (for 15 days before to 15 days after).

a) Sketch your graph on the axes below.



b) How many days before or after your 21<sup>st</sup> birthday does the spiritual cycle reach its minimum value?

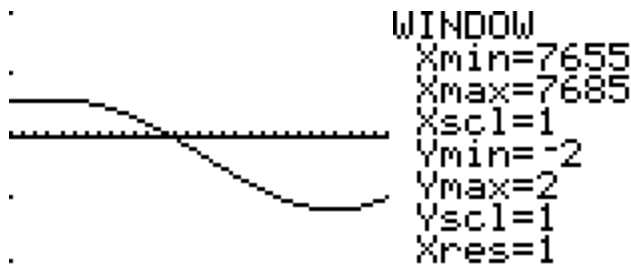
**The minimum value occurs on day 7671, or one day after your 21<sup>st</sup> birthday.**

3) Now, suppose that you wish to find the value of the sum of your spiritual cycle and your aesthetic cycle for the  $x$ th day of your life. Give an equation which models this sum.

Equation:  $y = \sin\left(\frac{2\pi}{53}x\right) + \sin\left(\frac{2\pi}{43}x\right)$



4) Now suppose that you would like to calculate the value of the sum of your spiritual cycle and your aesthetic cycle in the 30-day period surrounding your 21<sup>st</sup> birthday. Sketch your graph on the axes below.



5) Suppose that you have a friend who was born 90 days after you were. Write an equation which describes the value of his spiritual biorhythmic cycle for the  $x$ th day of *your* life.

Equation:  $y = \sin\left(\frac{2\pi}{53}(x-90)\right)$

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6) Now suppose that you would like to know the value of the sum of your spiritual cycle and your friend's spiritual cycle on the  $x$ th day of *your* life. Write an equation which models this sum.

Equation:  $y = \sin\left(\frac{2\pi}{53}x\right) + \sin\left(\frac{2\pi}{53}(x-90)\right)$

---

7) Use your calculator to graph your equation from Problem 6 for a period of 30 days surrounding your 21<sup>st</sup> birthday. How many days before or after your 21<sup>st</sup> birthday does this equation reach its minimum value?

**The minimum value of -1.166 occurs on day 7663 ( $x=7663.7506$ ), or 7 days before your 21<sup>st</sup> birthday.**

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